

ACTIVITY 2: STUDENT WORKSHEET



A. Testing the Small Angle Approximation

In this activity, you will test “the small angle approximation” in order to determine the limits over which it holds. The small angle approximation states: For small enough angles, the tangent of an angle is equal to the angle itself (when measured in radians). Or: $\tan \alpha = \alpha$ where α is the angle that you are measuring.

In the following table, convert the angles in degrees given in the first column to radians. Write your answers in column 2. Since there are 180 degrees in π radians, use the following conversion equation:

$$(\text{angle in degrees}) \times (\pi/180) = \text{angle in radians}$$

-or-

$$(\text{angle in degrees}) \times (0.01745) = \text{angle in radians}$$

Now find the tangent of the angle and write your answers in column 3 (if your calculator is set to degrees mode, use the numbers in the first column to calculate the tangent. Alternatively, you can use the numbers in the second column to find the tangent, if your calculator is set to radians mode).

In column 4, find the differences between the angle in radians (column 2) and the tangent of the angle (column 3).

In column 5, calculate the percentage difference using your numbers from column 4 and the original angle in radians (column 2), and the following formula:

$$\% \text{ difference} = \frac{\text{difference}}{\text{angle in radians}} \times 100$$

When you fill in the table, make sure you write out the numbers to *four* significant figures.

Angle (degrees)	Angle (radians)	Tangent (angle)	Difference	% Difference
0.5				
1				
2				
5				
10				
15				
20				
25				
30				
35				
40				

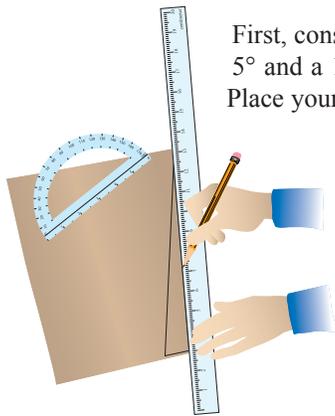
Name: _____ Period: _____ Date: _____



Question 1: One of the largest astronomical objects that you can see in the sky is the Moon, but it is still quite small - the full Moon is only 0.5 degrees across. Would the small angle formula give you a good measurement of the Moon's true size if you knew its distance?

Question 2: The active galaxies with the largest apparent angle in the sky are smaller than the apparent angle of the Moon. If we measure their distance, how accurate do you think the measure of their true size would be? _____

B. Measuring the Angular Size of a Person

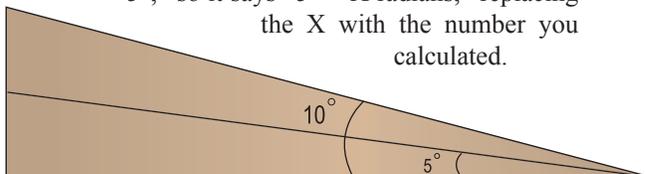


First, construct a template that measures a 5° and a 10° angle to use in the exercise. Place your stiff piece of cardboard in front of you so that one of the long edges is nearest you. Mark a point near the lower right hand corner on that long edge.

Place your protractor so that the hole in the bottom edge of the protractor is centered on the mark on the cardboard.

First, measure a 5° angle going off to the left hand side and mark it. Using a straightedge, connect the first mark to the second, creating a 5° angle with the bottom edge of the cardboard. Make the line as long as possible, and draw it dark enough to see well.

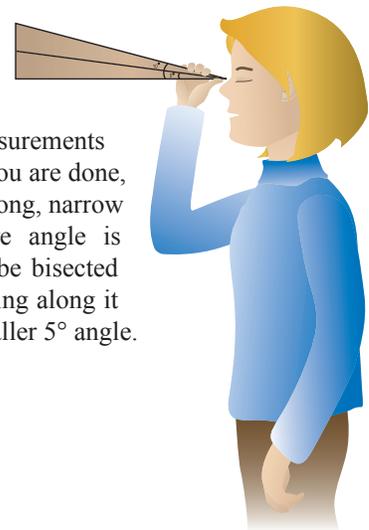
Now label the angle. Draw a little arc going from the bottom of the template up to the 5° line. Next to it, write " 5° ." Then convert that angle to radians and write that number next to where you wrote " 5° ," so it says " $5^\circ = X$ radians," replacing the X with the number you calculated.



Next, repeat this procedure using a 10° angle. You should now have two lines, one at 5° and the other at 10° , starting at the lower right corner of the

paper and going toward the upper left. Using scissors, carefully cut the cardboard along the 10° angle.

Make sure you cut the vertex of the angle carefully! If the narrow tip of the angle gets cut off, your measurements will be off. When you are done, you should have a long, narrow triangle. The entire angle is 10° , and it should be bisected by a dark line running along it that measures a smaller 5° angle.



You can check the accuracy of your template by measuring the 10° angle with your protractor again. It should be as close to 10° as possible, but no more than 0.5° off. If it's off by more than that, you'll need to either trim your template or make a new one.

Pick the roles each team member will perform in this activity: *Student A* will be using the template to measure angles, *Student B* will be measured, and *Student C* will be the measurer.

Student C: Using the meter stick, carefully measure the height of Student B to within a centimeter.

Question 3: Height of Student B:
_____ (cm)

Student A will now measure the angular size of Student B. Make sure you have enough room to do this! You'll need about 8-15 meters between them, so you may have to do this in a hallway or outside.

Students A and B: Start off by standing next to each other.

Student C: Mark the position of Student A on the floor/ground. (The mark should represent where Student A's eyes are, and not toes! Gauge where Student A's eyes are over the floor, or simply mark where the ankle is, which is roughly under the eyes.)

Student B: Start walking away from Student A.

Student A: Using the angle template, compare the size of Student B to the 10° angle on the template. (The best way to do this is to pinch the narrow end of the angle with your thumb and index finger, and hold it up to your face on the outside of your eye. That way, the vertex of the angle is aligned with your eye.) When Student B has walked far enough away that s/he appears to be the same size as the 10° angle, tell Student B to stop. Student B may need to move a bit closer or farther to adjust his or her angular size to match the template. If Student B appears smaller than the angle, tell Student B to move closer to you until Student B appears to be the same size as the end of the angle measure. If

Student B appears too large, tell Student B to move away. Match the angle as carefully as you can. Remember, mark the floor under the eyes, not the toes!

Student C: When Student B is at the right distance, mark this position.

Student C: With the meter stick or tape measure, measure the distance between Student A and Student B, to the nearest centimeter.

Question 4: Measured distance:
_____ (cm)

Calculate the height of Student B using the small angle formula: $\alpha = d/D$. In this equation, α is the 10° angle (but in radians!) that you used to position the student, lower case **d** is the height of Student B and upper case **D** is the distance that you have measured in question 4.

Question 5: Calculated height:
_____ (cm)

How close were you able to calculate the actual height? Subtract the calculated value from the measured value.

Question 6: Difference in height:
_____ (cm)

Calculate this difference as a percent:

The percent difference in distance =

$$\frac{(\text{measured} - \text{calculated})}{(\text{measured})} \times 100$$

Question 7: Percent difference:

How far away would Student B have to stand from Student A in order to have an angular measure of 0.5 arcminutes, the approximate resolution of GLAST? Remember there are 60 arcminutes in a degree, and you must convert the 0.5 arcminute angle to radians. Use the small angle formula to express your answer in kilometers, to the nearest 10 meters (0.01 km)

Question 8: Distance for 0.5 arcminutes: _____ (km)

Question 9: Do you think you would still be able to see Student B as more than a dot with your unaided eye from that distance?

C. Measuring the Angular Size of a Galaxy Using the Active Galaxies Poster

Student C: With a meter stick or metric ruler, measure the diameter of the disk along its longest dimension using the *middle* picture on the left of the poster. Measure to the nearest 0.1 centimeters.

Question 10: AG disk size: _____ (cm)

Student A: Move away from the poster until the gas disk in the middle left panel of the poster subtends an angle of 5° as measured with the cardboard angle template.

Student C: Mark the spot on the floor as in the previous exercise, and measure the distance from Student A to the poster.

Question 11: Measured disk distance (5°): _____ (cm)

Using your answer from question 11 and the small angle formula, calculate the size of the disk to the nearest 0.1 centimeters.

Question 12: Calculated disk size (5°): _____ (cm)

How accurate was your measurement? Calculate the percent difference between the measured and calculated sizes of the disk.

Question 13: Percent difference in disk sizes: _____

Using the small angle formula, calculate *how far* you would have to stand from the poster so that the disk would subtend 0.5 arcminutes - the resolution of the GLAST telescope. Express your answer in meters.

Question 14: Disk distance (0.5'): _____ (m)

Student C: With a meter stick or metric ruler, measure the radio lobe span of the AG in the *upper left corner* of the poster, from **radio lobe tip** to radio lobe tip.

Question 15: AG radio lobe size: _____ (cm)

Find the distance from the poster so that the radio lobes (from tip to opposite tip) subtend an angle of 5° .

Question 16: Measured radio lobe distance (5°): _____ (cm)