ACTIVE GALAXIES

An Educator’s Guide with Activities
in Science and Mathematics
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Busy educators sometimes have trouble finding ways to help their students feel the excitement of science in action. As a part of its educational effort, the NASA Education and Public Outreach group at Sonoma State University (SSU) has put together a series of activity presentations based on the science of one of NASA's exciting space missions: the Fermi Gamma-ray Space Telescope.

Since many students remember and understand better when they actively engage in manipulating the concepts about which they are learning, we have included several hands-on activities to help keep their interest and reinforce their comprehension and retention of the scientific concepts covered in the presentation of the mission. We have also included information about Fermi, what kind of objects it observes and why astronomers are interested in them. To help you determine when this project might be of most use to you in your science and/or math curriculum, we have included a matrix of the math and science standards covered. This introduction to the activities includes the answers to some frequently asked questions.

**What is Fermi?**

The Fermi Gamma-ray Space Telescope is a NASA satellite that was launched in 2008. Fermi is an Astrophysics Division mission within NASA's Science Mission Directorate. Astronomical satellites like Fermi are designed to explore the structure of the Universe, examine its cycles of matter and energy, and peer into the ultimate limits of gravity: black holes. Fermi detects gamma rays, the highest energy light in the electromagnetic spectrum. Fermi was built in collaboration between NASA, the U.S. Department of Energy, France, Germany, Italy, Japan, and Sweden. The project is managed from NASA's Goddard Space Flight Center in Greenbelt, Maryland.

**Where do gamma rays come from?**

Gamma-ray sources include black holes, pulsars, supernova remnants, and active galaxies. Active galaxies are a very common source of high-energy gamma rays detected by Fermi. Thousands of these sources are being studied during its mission.

**What instruments does Fermi use?**

There are two scientific instruments on board Fermi: the Large Area Telescope (LAT) and the Gamma-ray Burst Monitor (GBM). The LAT is the primary instrument, and surveys the sky in high-energy gamma-ray light. It also studies many individual sources of gamma rays. The GBM detects gamma-ray bursts, tremendous explosions coming from vast distances. These explosions are thought to signal the birth of black holes.

**What will my students learn from these activities?**

This series of activities uses active galaxies – distant galaxies with supermassive black holes in their cores – as an engagement to teach basic concepts in physical science and mathematics. The mapping of the activities to the relevant national standards is shown by the matrix on page 4, and a more detailed listing of the standards met by each activity can be found in pages 31 through 32.

**How are these activities organized?**

Page 5 has a general introduction to active galaxies. Depending on the age and ability of your students, you may need to tell them about this information, have them read it, or have a few advanced students put together a presentation to the rest of the class based on this introduction and other sources they may find. With this introduction, try to convey
the excitement of the scientists when they first discovered interesting phenomena related to active galaxies, such as the super-massive black holes at their cores, and the high-energy jets of particles and light that are sometimes emitted.

Each activity has some or all of the following components to help you make it an exciting learning experience for the students:

1) Science concepts and estimated time. (Note: time varies significantly for different age groups and levels of science understanding.)

2) Background information specific to this activity. (See the suggestions above for possible ways to present this background information.)

3) The “essential question” asked by the activity. Take the time to help students understand that scientists ask questions. Each activity states the essential question that this activity is designed to answer, or to help the student explore.

4) The materials needed to complete the activity.

5) A list of abbreviations used and possible additional notes to the teacher.

6) The specific learning objectives of this activity.

7) The step-by-step procedures to be followed for the most efficient and effective use of the activity.

8) An assessment rubric for the activity. It is important that before you start an activity you have a clear understanding of what constitutes a successful activity. This assessment rubric suggests ways to evaluate your students’ work in mastering the activities’ objectives.

9) Transfer activities: One of our goals in science is to help students see science and scientific concepts as tools to be used throughout their lives, not just as a small part of their education. Including transfer activities after the activity is completed will not only reinforce the specific objectives, but also help your students learn to apply scientific concepts to their “real lives.”

10) Suggested extension and reflection activities. These help the student follow up the activity with comprehension exercises so that they better assimilate the information, and use the concepts they have learned to better understand phenomena in everyday life.

11) Lesson adaptations that will help you cope with special needs students.

12) An answer key that provides you with the answers to the questions given to the students, and that will help you evaluate the products the students may produce as a part of this activity.

13) Student worksheets that contain the information and directions necessary for the student to complete the activities. NOTE: giving the students the work sheet without the appropriate background information and procedures will not only decrease the learning of the students, it may also cause frustration and feelings of inadequacy to master science principles.

14) Detailed Standards lists that explain how each activity meets the specific national science and mathematics standards.

15) A glossary that briefly defines each term with which the student may be unfamiliar (these appear in bold letters the first time they are used in the activity).

16) A list defining any acronyms used.

17) A resource list which will help you find more information about the topics in the activities.

Who developed these activities?

The activities and the poster that describes active galaxies have been developed as part of the NASA Education and Public Outreach (E/PO) Program at Sonoma State University, under the direction of Professor Lynn Cominsky. This guide and the accompanying poster can be downloaded from: http://fermi.sonoma.edu/teachers/agn.php

Contributors to this education unit also include Dr. Philip Plait, Lynda Williams, Sarah Silva, Michelle Curtis, Aurore Simonnet, Tim Graves, and Dr. Mary Garrett. We would also like to acknowledge input from Kara Granger and Dr. Laura Whitlock to an earlier version of the first activity, and helpful comments from Sharon Janulaw, Teena Della, Dr. Christine Royce, Dr. William Keel, Dr. Greg Madejski, Dr. Tom Arnold, Rae McIntyre, Bruce Hemp, and Tom Estill.
The activities in this booklet conform to these National Science and Mathematics Education Standards (NSES and NCTM). Detailed standards met by each activity are listed on pages 31-32.

<table>
<thead>
<tr>
<th>Content Standard</th>
<th>Science Standards</th>
<th>Math Standards</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>Building Perspectives with Active Galaxies</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Zooming In on Active Galaxies</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Light Travel Time and the Size of Active Galaxies</td>
<td>✓</td>
<td>✓</td>
</tr>
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</table>
A galaxy is a system of stars, gas, and dust bound together by their mutual gravity. A typical galaxy has billions of stars, and some have trillions. Although they come in many different shapes, the basic structure is the same: a dense core of stars called a ‘nucleus’ surrounded by stars and gas. Normally, the core of a disk or elliptical galaxy is small, relatively faint, and composed of older, redder stars. However, in some galaxies the core is intensely bright, shining with power equivalent to trillions of suns, easily outshining the rest of the light of the galaxy combined. A galaxy that emits such tremendous amounts of energy is called an active galaxy, or AG for short. Active galaxies are actually rare, but so bright they can be seen clear across the visible universe.

It is believed that at the center of these bright galaxies lies a supermassive black hole millions or even billions of times the mass of our Sun (Figure 1). As matter falls toward the black hole, it forms an accretion disk, a flattened disk of material swirling around the black hole. Friction and magnetic forces inside the disk heat it to millions of degrees, and it glows brightly nearly all the way across the electromagnetic spectrum, from radio waves up to X-rays. Although our own Milky Way Galaxy has a central supermassive black hole, it is not an active galaxy. For reasons currently unknown, the black hole at the center of our Galaxy is quiescent, or inactive, as are most present-day galaxies.

Although the physics underlying the phenomenon is not well-understood, it is known that in some cases long jets of matter and energy streak away from the core at speeds near that of light. The jets are highly collimated (meaning they retain their narrow focus over vast distances) and are emitted in a direction perpendicular to the disk.

Eventually, they slow to a stop due to friction with gas well outside the galaxy, forming giant clouds of matter that radiate strongly at radio wavelengths (radio lobes; Figure 2). In addition, surrounding the accretion disk is a torus (donut) of molecular material. From certain viewing angles, this torus can obscure observations of the black hole and accretion disk.

There are many types of active galaxies. Initially, when astronomers were first studying them, it was thought that the different types of AGs were fundamentally different objects. Now astronomers generally (but not universally) accept the unified model of AGs, meaning that most or all AGs are actually just different versions of the same object. Many of the apparent differences between types of AGs are due to viewing the AG at different orientations with respect to the disk, or due to observing the AG in different wavelengths of light.

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Fig. 1 Artist’s illustration of an active galaxy

Fig. 2 Double-lobed radio galaxy NGC4261

Ground-Based Radio Image  Hubble Image of a Gas and Dust Disk

386 Arc Seconds 88,000 LIGHT-YEARS  1.7 Arc Seconds 400 LIGHT-YEARS
Background Information:

The type of active galaxy we see depends on the way that we see it. If we see the accretion disk and gas torus edge on, the galaxy is called a radio galaxy. The torus of cool gas and dust blocks most of the radiation from the inner black hole and its nearby environment, so the most obvious features are the radio emitting jets and giant lobes well outside the galaxy.

If the disk is tipped slightly to our line of sight, we can see higher-energy light from the accretion disk inside the gas torus in addition to the lower energy radio waves. This kind of AG is called a Seyfert galaxy (named after American astronomer Carl Seyfert, who first catalogued these galaxies in 1943). It looks much like a normal galaxy but with a very bright core, and may be giving off high-energy photons like X-rays. If the galaxy is very far away from us, we may see the core as a star-like object even if the fainter surrounding galaxy is undetected. In this case, the galaxy is called a quasar, which is short for quasi-stellar radio source (so-named because the first ones discovered appeared to be star-like through a telescope, but emitted copious radio waves, unlike “normal” stars). The first quasar to be discovered, dubbed 3C273, was found to be a galaxy at a very large distance by astronomer Martin Schmidt in 1963.

If the tip angle is 90 degrees, we can be looking straight down a jet. This type of active galaxy is called a blazar. From blazars we see very high-energy gamma ray photons. The first blazar to be discovered, BL Lac (and after which we get the term “blazar”) was found in 1926 to change in brightness, but was thought to be a normal star! It wasn’t until the late 1970s that its galactic nature was truly revealed.

In sum, the basic components of an active galaxy are: a supermassive black hole core, an accretion disk surrounding it, and a torus of gas and dust, and in some (but not all!) highly focused jets of matter and energy. The type of active galaxy we see depends on the way we see the galaxy: radio galaxies, Seyferts, quasars and blazars.

Science Concepts:

- There are different components in active galaxies.
- Different viewing angles lead to dramatic differences in the appearance of simple objects.

Essential Question:
What do active galaxies look like when viewed from different directions?

Brief overview:
Students will explore how position and perception affect our understanding of phenomena by making a model of an active galaxy.
Materials for each group of 2 or 3 students:

- Styrofoam ball (1.5 inch or 4 cm in diameter)
- 2 toothpicks
- 1 lightweight sheet of construction paper
- 1 heavier weight sheet of construction paper
- 3 pages of blank or bond white paper
- scissors
- compass
- ruler
- cellophane tape
- colored pencils or pens

Objective:
Students will build a model of an active galaxy. From this, they will learn about the geometry of the components of the galaxy and understand that different viewing angles lead to dramatically different appearances of the galaxies.

Procedure:

1. Introduce the activity by using the information in the Introduction to Active Galaxies (page 5) and in the activity Background Information (page 6).

2. Discuss these questions with the students before starting the activity:
Do objects look the same from all angles? Are they recognizable from all angles? For example, it’s not difficult to recognize a book from almost any viewing angle. But is every object like that? How do we recognize objects if viewed from unfamiliar angles? How do we categorize unfamiliar objects if seen from different angles?

3. Explain to the students that they will be building a model of an active galaxy from a styrofoam ball, construction paper, toothpicks and tape. This model will help them answer the questions on the worksheet. The model building can be done in groups of two or three.

4. Have the students make observations of the model, then sketch them as viewed from different angles. This should be done individually.

Assessment: The students’ work can be assessed using the following rubric:

<table>
<thead>
<tr>
<th>Points</th>
<th>Points</th>
<th>Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>➔ All calculations are correct</td>
<td>➔ Two calculations are incorrect, or two drawings drawn/labeled incorrectly, or model built incorrectly, or two descriptions are incorrect, or some combination</td>
<td></td>
</tr>
<tr>
<td>➔ Model is assembled correctly and neatly</td>
<td>➔ Model is assembled correctly and neatly</td>
<td></td>
</tr>
<tr>
<td>➔ Drawings are complete and neatly done</td>
<td>➔ Description of different views are accurate</td>
<td></td>
</tr>
<tr>
<td>➔ Description of different views are accurate</td>
<td>➔ Description of different views are accurate</td>
<td></td>
</tr>
<tr>
<td>➔ One calculation is incorrect, or one drawing is drawn/labeled incorrectly, or one description is incorrect</td>
<td>➔ Drawings or descriptions are incorrect</td>
<td></td>
</tr>
<tr>
<td>➔ No work turned in</td>
<td>➔ No work turned in</td>
<td>➔ No work turned in</td>
</tr>
</tbody>
</table>
Extension Activities:

Compare and contrast the students’ drawings to those on the Fermi Active Galaxies Poster for the different viewing angles. Discuss how this activity changes their perspectives about how they view and interpret what they see in the Universe.

Transfer Activities:

Have the students examine and draw everyday objects from different angles. Pass the drawings around to other students and see if they can identify the object, especially if the viewing angle is unfamiliar.

Lesson Adaptations:

Visually impaired students may have difficulty constructing the models and drawing them. Put the model in their hands, and let them note by touch how the model feels different if they can only access one part of it at a time (for example, a single cone/jet). They can examine how the model feels different if they keep their hand flat, fingers extended, and can only touch the model that way. In that example, the opening of the cone will feel like a circle, and the torus will feel flat. Have them describe how limiting their ability to touch the model limits their ability to identify its parts.

Answer Key for “Building Perspectives with Active Galaxies”

1) 1.5 inches or 4 cm

2) If the conical paper jet is properly constructed, it will obscure the styrofoam ball (black hole).

3) For a 4 cm diameter styrofoam ball, the minimum width of the strip should be 4 cm (1.5 inches).

4) If D is the outer diameter of your disk, then the circumference of the torus is \( \pi D \). For an 8 cm disk, the circumference is then \( 8 \pi \approx 25.1 \) cm

5) This answer will depend on the student. In general, looking straight down the jets will hide the black hole, accretion disk and torus. From an angle, the black hole, accretion, disk and torus will be visible, and the jets will be elongated. The far jet may be difficult to see. From the side, the black hole and accretion disk are again hidden, this time by the torus. Both jets will be visible and about the same size.
A. Making a Model of an Active Galaxy

The styrofoam ball represents the black hole at the center of the active galaxy. In the next few steps, you are going to make an accretion disk, cones that represent the jets of the active galaxy, and the torus that surrounds the accretion disk.

**1. Accretion Disk:** Measure the diameter of your ball, just to be sure of the size. Note the size both in inches and centimeters here (1 in = 2.54 cm)

**Question 1:** Diameter: ____________ (in) ____________ (cm)

With your compass, draw a circle with the diameter of your styrofoam ball on the heavier weight construction paper. Using the same center, draw a concentric circle with twice the diameter. Cut the large circle and then cut out the inner circle to make an annular disk (a ring with a hole in the center). Gently work the styrofoam ball through the inner circle of your annular disk until it is exactly in the middle. This is your accretion disk! You can draw a swirling vortex on it and label “accretion disk” or other appropriate descriptive words that describe the properties of the disk.

**2. Jets:** With your compass, draw a circle 20 cm in diameter on the lightweight construction paper. With your ruler, draw 2 perpendicular lines through the center as shown. Cut the circle out and cut it in half. Each semicircle will be rolled into a cone to make a jet. Hold the half circle so that the drawn line is on the inside of the cone. Curl up the inner edge of the paper to the drawn line to make your cone. Tape the outside edge.

Make sure the apex of your cone is nice and tight so you can stick it to the styrofoam ball with a toothpick. When you have made both jets, attach them to the styrofoam ball by piercing the apex with a toothpick and sticking the cones symmetrically on the ball above and below the accretion disk.
B. Drawing the Active Galaxy Model from Different Perspectives

View your active galaxy model from the following three views:

1. Down a jet
2. At an angle to the jet (not 90 degrees!)
3. 90 degrees from the jet

9. Wrap around a strip of paper
You've just made an active galaxy!

Question 5: On separate pieces of paper, sketch what you see from those three angles. Describe how they are alike and how they are different.

Attach your drawings to this worksheet before you hand in your work.
Background Information

The appearance of a distant object and the amount of detail we can see in it is directly related to how far away it is from us - the closer an object is, the larger it appears and the easier it is to see details; the farther away it is, the smaller it appears and the harder it is to see details. Although active galaxies are often hundreds of thousands of light years in size, their great distances make them challenging to observe in detail. In this activity we explore how the distance to an active galaxy affects how we see it.

Let’s imagine distance is not a factor and take a fantasy flight into a cartoon active galaxy (AG) that has jets. The long view of the AG is dominated by lobes of radio emission caused by highly focused twin jets of matter streaming out from the galactic nucleus (Fig. 4). Closer in, a torus of dust and gas can be seen orbiting outside a flatter disk of gas (Fig. 5). In an extreme close-up, the invisible black hole in the center is surrounded by a flat accretion disk of rapidly orbiting material. Jets are emitted at right angles from the plane of the disk, driven by physics still not well understood (Fig. 6).

Now let’s consider a real AG: the double-lobed radio galaxy NGC 4261, which is located approximately 100 million light years from Earth. The Hubble Space Telescope and ground-based radio images (Figure 1, on page 5) of NGC 4261 shows radio lobes that span some 100,000 light years out from a spiral-shaped disk of gas and dust 400 light years in diameter. Presumably, a small but supermassive black hole “engine” lies at the center of the nucleus with a mass 1.2 billion times the mass of the Sun and contained in a space about the size of the solar system - about 6 billion kilometers. Imagine that!

How much detail can we actually see with the Hubble Space Telescope (HST) in a galaxy 100 million light years away? The smallest objects HST can discern have an angle of about 0.1 arcseconds (about 0.00005 times the width of the full Moon). Using the small angle approximation (see activity on page 12) this angular diameter translates into a linear size of 460 trillion kilometers at the distance of NGC 4261! This is over 100,000 times larger than the effective size of the black hole.

That’s why we can’t directly see the black hole at the core of the galaxy. Rather, we infer the existence and the properties of the black hole indirectly by observing the effect it has on the gas and stars surrounding it.

The Fermi LAT, designed to detect gamma rays from active galaxies, has a resolution of roughly 0.5 arcminutes (about 1/60th the width of the full Moon). Although no current detection technology exists that can see the central black hole in an active galaxy, Fermi is able to detect gamma rays produced by the jets with unprecedented sensitivity. Such data help us understand the physical processes that are going on in the nucleus and the supermassive black hole engine that fuels it.

Essential Question:
What do active galaxies look like when viewed from different distances?

Science Concepts:
• The small angle approximation has limits.
• The angular size of an object depends on its distance and its physical diameter.

Brief overview:
Students will test the small angle approximation and construct a template that measures 5° and 10° angles.
We imagine the sky to look like a giant hollow sphere surrounding the Earth. It really isn't, but real objects are so far away that we have no real perception of distance, and so when we look at stars in the sky our brain interprets them as being infinitely far away. This gives the sky an illusion of being a hollow sphere with us at the center.

When astronomers measure the size of objects in the sky or the distance between them, they use angular size. A circle drawn all the way around the sky is divided into 360 degrees, so two objects on opposite sides of the sky are said to be 180 degrees apart. An object that spans $\frac{1}{4}$ of the sky's circumference would be 90 degrees across, and so on. The disk of the Moon in these units has an angular diameter of 0.5 degrees. Degrees are further subdivided into 60 arcminutes (the term “arc” is to help distinguish this from a measure of time, and the unit is abbreviated as a single tick mark: ‘) and each arcminute is further divided into 60 arcseconds (abbreviated with a double tick mark: ”). The Moon can thus be described as having an angular diameter of 30 arcminutes, or 1800 arcseconds. Most distant astronomical objects such as galaxies are fractions of an arcsecond to a few arcminutes in angular extent.

The relationship between angular diameter, distance, and appearance is shown below: the closer an object is, the larger the angle it covers and the larger it appears; the farther away an object is, the smaller the angle it covers and the smaller it appears.

Mathematically, angular diameter, linear diameter, and distance can be combined in an extremely useful and simple equation called the small angle approximation. As seen in the figure below, the angular diameter, $\alpha$, depends on the distance to the object, $D$, and its actual linear diameter, $d$, according to:

$$\tan(\alpha/2) = d / 2D$$

The students will show in this part of the unit, for very small values of $\alpha$ measured in radians, $\tan(\alpha) = \alpha$. Using this approximation, the equation relating distance and linear size simplifies further to

$$\alpha/2 = d / 2D \quad \text{or more simply} \quad \alpha = d/D$$
In the small angle approximation, if any two of the quantities are known, the third can be calculated. In astronomy, the angular diameter is usually measured directly and the equation is used to calculate the distance or the physical diameter of the object. Since distances to astronomical objects are usually much larger than their linear sizes, this approximation is of great use in all branches and at all levels of astronomy!

The following table provides a summary of the unit conversions discussed above, along with some other abbreviations that will be used later in the unit. It may be useful for your students to have access to this table.

<table>
<thead>
<tr>
<th>Abbreviations</th>
<th>Conversion Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha)</td>
<td>(\text{Light year to meters: } 1 \text{ ly} = 9.5 \times 10^{15} \text{ m})</td>
</tr>
<tr>
<td>cm</td>
<td>(\text{Meters to centimeters: } 1 \text{ m} = 100 \text{ cm})</td>
</tr>
<tr>
<td>°</td>
<td>(\text{Kilometers to meters: } 1 \text{ km} = 1000 \text{ m})</td>
</tr>
<tr>
<td>m</td>
<td>(\text{Radians to degrees: } 1 \text{ rad} \times \frac{180^\circ}{\pi \text{ rad}} = 57.3^\circ)</td>
</tr>
<tr>
<td>mm</td>
<td></td>
</tr>
<tr>
<td>%</td>
<td></td>
</tr>
<tr>
<td>rad</td>
<td></td>
</tr>
<tr>
<td>s</td>
<td></td>
</tr>
</tbody>
</table>

\[\alpha\] angular diameter  
\[\text{cm}\] centimeters  
\[\text{°}\] degrees  
\[D\] diameter  
\[\text{ly}\] light year  
\[m\] meters  
\[\text{mm}\] millimeters  
\[\%\] percentage  
\[\text{rad}\] radian  
\[s\] second

**Note for the teacher:** Just like with hands, some people are right-eyed and others left-eyed. The instructions for the template construction below assume the student will be using their right eye. When the students are constructing the template, they can reverse left and right in the directions if they prefer to use their left eye.

**Materials for each group of 3 students:**
- Fermi Active Galaxies Poster
- protractor
- stiff cardboard
- meter stick or tape measure
- pencil
- scissors
- calculator

**Objectives:**
Students will be able to accurately use and understand the small angle formula, and demonstrate this by filling out a table.
Students will be able to construct a template and use it to correctly measure the angular size of a person.
Students will be able to use the Active Galaxies Poster to measure the angular size of a galaxy.
Procedure:

1. Introduce the activity by reviewing the information in the Introduction to Active Galaxies (page 5) and the activity Background Information (page 11).

2. Discuss these questions with the students: Can they recognize their friend’s face from across the room? How about from across a football field? How far away can someone be and still be recognizable?

3. Explain to the students that they will be doing a series of three activities that will teach them how the angular size of objects changes with distance.

4. The small angle activity (Part A) should be done individually. The other two activities (Parts B and C) should be done in teams of three people each. Part A can be eliminated if time is not available.

5. For Part C, “Measuring the Size of a Galaxy Using the Active Galaxy Poster,” you will need to affix the poster to a wall, and make sure the students have about 5 meters of clear space between them and the poster. You can mount the poster at the end of a hallway or on the wall of a large classroom.

Assessment:

There are three separate activities that comprise this unit. They can be assessed using the common rubric below either individually or as a group.

Points
- All calculations are correct: 4 Points
- One calculation is incorrect, or off by more than 10%: 3 Points
- Two calculations are incorrect: 2 Points
- More than two calculations are incorrect: 1 Point
- No work turned in: 0 Points

Transfer Activities:

Students can test the limits of their vision and measure angles of familiar objects. For example, the width of a finger held at arm’s length is typically 1–2 degrees. This can be calibrated by having them mark their angle template in degrees from 1–10, and using it to measure the width of their fingers held at arm’s length. By comparing the known size of an object (say, the length of a car) to the width of their finger, they can approximate the distance to the object using the small angle formula.

Extension Activities:

Derive the small angle formula more rigorously, following the rules of trigonometry. Given that the Moon is 3500 km in diameter and 0.5º in angular size, have the students calculate the distance to the Moon. Have them look up the correct answer and compare it to their calculated answer.

Reflection Activities:

Where else would the small angle formula be useful? Examples: in surveying, hunting, sports, or anywhere else where gauging the distance would be necessary.
Lesson Adaptations:

If the student is visually impaired, and cannot measure the angles using the template in Activity 2, s/he can be the person whose height is measured, or can use the meter stick to measure distances. If the student is mobility-impaired, s/he can be the person whose height is measured (Student B).

Answer Key for “Zooming In on Active Galaxies”:

Part A:

<table>
<thead>
<tr>
<th>Angle (degrees)</th>
<th>Angle (radians)</th>
<th>Tangent (angle)</th>
<th>Difference</th>
<th>% Difference</th>
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</thead>
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<td>0.1745</td>
<td>0.1763</td>
<td>0.0018</td>
<td>1.032</td>
</tr>
<tr>
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<td>0.2618</td>
<td>0.2679</td>
<td>0.0066</td>
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</tr>
<tr>
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<td>0.3491</td>
<td>0.3640</td>
<td>0.0149</td>
<td>4.270</td>
</tr>
<tr>
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<td>0.4363</td>
<td>0.4663</td>
<td>0.0300</td>
<td>6.870</td>
</tr>
<tr>
<td>30.00</td>
<td>0.5236</td>
<td>0.5774</td>
<td>0.0538</td>
<td>10.27</td>
</tr>
<tr>
<td>35.00</td>
<td>0.6109</td>
<td>0.7002</td>
<td>0.0893</td>
<td>14.63</td>
</tr>
<tr>
<td>40.00</td>
<td>0.6981</td>
<td>0.8391</td>
<td>0.1410</td>
<td>20.19</td>
</tr>
</tbody>
</table>

1. The small angle approximation works well for the full Moon.
2. Yes, the small angle approximation should work very well.

Part B: Answers for questions 3 through 8 will depend on each student’s height. Below is a reference table with distances given the students’ height.

<table>
<thead>
<tr>
<th>Student Height (cm)</th>
<th>Distance (cm) for 0.17 radians=10°</th>
<th>Distance (km) for 0.0001 radians = 0.5 arcmin</th>
</tr>
</thead>
<tbody>
<tr>
<td>140</td>
<td>802.1</td>
<td>9.626</td>
</tr>
<tr>
<td>150</td>
<td>859.4</td>
<td>10.31</td>
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<tr>
<td>160</td>
<td>916.7</td>
<td>11.00</td>
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<tr>
<td>170</td>
<td>974.0</td>
<td>11.69</td>
</tr>
<tr>
<td>180</td>
<td>1031</td>
<td>12.38</td>
</tr>
<tr>
<td>190</td>
<td>1089</td>
<td>13.06</td>
</tr>
<tr>
<td>200</td>
<td>1146</td>
<td>13.75</td>
</tr>
</tbody>
</table>
9.  On average, the typical human eye can see objects about $1/60^{th}$ of a degree across, so the answer to this question is "no."

10.  13.0 centimeters. Note the significant figures should reflect 0.1 cm accuracy.

11.  The distance should be 149 cm. This will depend on their measuring accuracy.

12.  This will depend on their distance measurement, but should be close to the actual disk size of 13 cm.

13.  This will depend on their accuracy. They should be within 10% or so of the measured size.

14.  893.8 meters.

15.  17.5 centimeters. Note the significant figures should reflect 0.1 cm accuracy.

16.  The distance should be 200.5 cm, and will depend on their measuring accuracy.

17.  This will depend on their distance measurement, but should be close to the actual lobe size of 17.5 cm.

18.  This will depend on their accuracy. They should be within 10% or so of the measured size.

19.  1203.2 meters.

20.  446.9 meters.

21.  1.38 million light years.

22.  $100 \text{ million} / 1.38 \text{ million} = 73$, so the magnification would be 73X.
A. Testing the Small Angle Approximation

In this activity, you will test “the small angle approximation” in order to determine the limits over which it holds. The small angle approximation states: For small enough angles, the tangent of an angle is equal to the angle itself (when measured in radians). Or: \( \tan \alpha = \alpha \) where \( \alpha \) is the angle that you are measuring.

In the following table, convert the angles in degrees given in the first column to radians. Write your answers in column 2. Since there are 180 degrees in \( \pi \) radians, use the following conversion equation:

\[
\text{(angle in degrees)} \times \left(\frac{\pi}{180}\right) = \text{angle in radians}
\]

- or -

\[
\text{(angle in degrees)} \times (0.01745) = \text{angle in radians}
\]

Now find the tangent of the angle and write your answers in column 3 (if your calculator is set to degrees mode, use the numbers in the first column to calculate the tangent. Alternatively, you can use the numbers in the second column to find the tangent, if your calculator is set to radians mode).

In column 4, find the differences between the angle in radians (column 2) and the tangent of the angle (column 3).

In column 5, calculate the percentage difference using your numbers from column 4 and the original angle in radians (column 2), and the following formula:

\[
\% \text{ difference} = \frac{\text{difference}}{\text{angle in radians}} \times 100
\]

When you fill in the table, make sure you write out the numbers to four significant figures.

<table>
<thead>
<tr>
<th>Angle (degrees)</th>
<th>Angle (radians)</th>
<th>Tangent (angle)</th>
<th>Difference</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
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<td>2</td>
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<td></td>
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<tr>
<td>5</td>
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<td>35</td>
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</tr>
<tr>
<td>40</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
B. Measuring the Angular Size of a Person

First, construct a template that measures a 5° and a 10° angle to use in the exercise. Place your stiff piece of cardboard in front of you so that one of the long edges is nearest you. Mark a point near the lower right hand corner on that long edge.

Place your protractor so that the hole in the bottom edge of the protractor is centered on the mark on the cardboard. First, measure a 5° angle going off to the left hand side and mark it. Using a straightedge, connect the first mark to the second, creating a 5° angle with the bottom edge of the cardboard. Make the line as long as possible, and draw it dark enough to see well.

Now label the angle. Draw a little arc going from the bottom of the template up to the 5° line. Next to it, write “5°.” Then convert that angle to radians and write that number next to where you wrote “5°,” so it says “5° = X radians,” replacing the X with the number you calculated.

Next, repeat this procedure using a 10° angle. You should now have two lines, one at 5° and the other at 10°, starting at the lower right corner of the paper and going toward the upper left. Using scissors, carefully cut the cardboard along the 10° angle.

Make sure you cut the vertex of the angle carefully! If the narrow tip of the angle gets cut off, your measurements will be off. When you are done, you should have a long, narrow triangle. The entire angle is 10°, and it should be bisected by a dark line running along it that measures a smaller 5° angle.
You can check the accuracy of your template by measuring the 10° angle with your protractor again. It should be as close to 10° as possible, but no more than 0.5° off. If it’s off by more than that, you’ll need to either trim your template or make a new one.

Pick the roles each team member will perform in this activity: Student A will be using the template to measure angles, Student B will be measured, and Student C will be the measurer.

**Student C:** Using the meter stick, carefully measure the height of Student B to within a centimeter. 
**Student A** will now measure the angular size of Student B. Make sure you have enough room to do this! You’ll need about 8-15 meters between them, so you may have to do this in a hallway or outside.

**Students A and B:** Start off by standing next to each other.

**Student C:** Mark the position of Student A on the floor/ground. (The mark should represent where Student A’s eyes are, and not toes! Gauge where Student A’s eyes are over the floor, or simply mark where the ankle is, which is roughly under the eyes.)

**Student B:** Start walking away from Student A.

**Student A:** Using the angle template, compare the size of Student B to the 10° angle on the template. (The best way to do this is to pinch the narrow end of the angle with your thumb and index finger, and hold it up to your face on the outside of your eye. That way, the vertex of the angle is aligned with your eye.) When Student B has walked far enough away that s/he appears to be the same size as the 10° angle, tell Student B to stop. Student B may need to move a bit closer or farther to adjust his or her angular size to match the template. If Student B appears smaller than the angle, tell Student B to move closer to you until Student B appears to be the same size as the end of the angle measure. If Student B appears too large, tell Student B to move away. Match the angle as carefully as you can. Remember, mark the floor under the eyes, not the toes!

**Student C:** When Student B is at the right distance, mark this position.

**Student C:** With the meter stick or tape measure, measure the distance between Student A and Student B, to the nearest centimeter.

**Question 4:** Measured distance: ____________________ (cm)

Calculate the height of Student B using the small angle formula: \( \alpha = \frac{d}{D} \). In this equation, \( \alpha \) is the 10° angle (but in radians!) that you used to position the student, lower case \( d \) is the height of Student B and upper case \( D \) is the distance that you have measured in question 4.

**Question 5:** Calculated height: ____________________ (cm)

How close were you able to calculate the actual height? Subtract the calculated value from the measured value.

**Question 6:** Difference in height: ____________________ (cm)

Calculate this difference as a percent:

The percent difference in distance = 
\[
\frac{(\text{measured} - \text{calculated})}{\text{(measured)}} \times 100
\]

**Question 7:** Percent difference: ____________________
How far away would Student B have to stand from Student A in order to have an angular measure of 0.5 arcminutes, the approximate resolution of Fermi? Remember there are 60 arcminutes in a degree, and you must convert the 0.5 arcminute angle to radians. Use the small angle formula to express your answer in kilometers, to the nearest 10 meters (0.01 km).

**Question 8:** Distance for 0.5 arcminutes: ____________ (km)

**Question 9:** Do you think you would still be able to see Student B as more than a dot with your unaided eye from that distance? ______________

**Question 11:** Measured disk distance (5°): ______________ (cm)

Using your answer from question 11 and the small angle formula, calculate the size of the disk to the nearest 0.1 centimeters.

**Question 12:** Calculated disk size (5°): ______________ (cm)

How accurate was your measurement? Calculate the percent difference between the measured and calculated sizes of the disk.

**Question 13:** Percent difference in disk sizes: ______________

Using the small angle formula, calculate how far you would have to stand from the poster so that the disk would subtend 0.5 arcminutes - the resolution of the Fermi telescope. Express your answer in meters.

**Question 14:** Disk distance (0.5°): ______________ (m)

**Question 15:** AG radio lobe size: ______________ (cm)

Find the distance from the poster so that the radio lobes (from tip to opposite tip) subtend an angle of 5°.

**Question 16:** Measured radio lobe distance (5°): ______________ (cm)

C. Measuring the Angular Size of a Galaxy

Using the Active Galaxies Poster

**Student C:** With a meter stick or metric ruler, measure the diameter of the disk along its longest dimension using the middle picture on the left of the poster. Measure to the nearest 0.1 centimeters.

**Question 10:** AG disk size: ______________ (cm)

**Student A:** Move away from the poster until the gas disk in the middle left panel of the poster subtends an angle of 5° as measured with the cardboard angle template.

**Student C:** Mark the spot on the floor as in the previous exercise, and measure the distance from Student A to the poster.

**Student C:** With a meter stick or metric ruler, measure the radio lobe span of the AG in the upper left corner of the poster, from radio lobe tip to radio lobe tip.

**Question 15:** AG radio lobe size: ______________ (cm)

Zooming in p.4
What are the limits of your own vision? The average human eye can just barely distinguish two objects that are 1 arcminute apart. (Your own vision may vary from this.) Using the small angle formula, determine how far away the poster would have to be in order for you to barely see the disk as more than a dot. Express your answer in meters.

**Question 17: Calculated radio lobe size (5°):** _______________ (cm)

Calculate the percent difference in your measured versus calculated sizes for the radio lobes.

**Question 18: Percent difference in radio lobe sizes:** __________

How far would you have to stand from the poster so the radio lobes subtend 0.5 arcminutes? Express your answer in meters.

**Question 19: Radio lobe distance (0.5°):** _______________ (m)

What are the limits of your own vision? The average human eye can just barely distinguish two objects that are 1 arcminute apart. (Your own vision may vary from this.) Using the small angle formula, determine how far away the poster would have to be in order for you to barely see the disk as more than a dot. Express your answer in meters.

**Question 20: Limit Distance:** _______________ (m)

The disk of NGC 4261 (see image below) is 400 light years in diameter. Use the small angle formula to determine the maximum distance (in light years) at which you could see this disk as more than a dot with your naked eye. (NOTE: To see something as more than a dot, its angular size must be at least 1 arcminute).

**Look again at your answer for Question 9. Did you answer the question correctly?**

How far would you have to stand from the poster so the radio lobes subtend 0.5 arcminutes? Express your answer in meters.

**Question 21: NGC 4261 Distance (1°):** _______________

Compare your answer to the actual distance to NGC 4262 of 100 million light years. What is the ratio of these distances?

**Question 22: Ratio:**

\[
\frac{\text{actual}}{\text{actual for } 1°}
\]
Background Information

Active galaxies do not usually emit a steady stream of light. Instead, their brightness can change dramatically over the span of weeks, days and even minutes. Fermi detects this variability from thousands of AGs with great accuracy. Amazingly, this variability can tell astronomers quite a bit about the AG itself, including its size!

Astronomers can find the size of the AG emitting region using a clever technique. It involves knowing the speed of light, and being able to set limits on how quickly an object can change its brightness.

Imagine an object that measures one light-week across as in the figure below. Suppose that the entire object emits a brief flash of light. Photons from the part of the object nearest to the Earth arrive at our telescopes first. Photons from the middle of the object arrive at the Earth some time later. Finally, light from the far side of the object arrives after a measurable time difference from the arrival of the first photons. Although the object emitted a sudden flash of light, we observe a gradual increase in brightness that lasts a full week, from the first recorded incident. In other words, the flash is stretched out over a time interval equal to the difference in the light travel time between the nearest and most remote observable regions of the object.

If an object is 1 light-year in diameter, it will take 1 year longer for the signal from the far side of the object to be detected than the signal from the near side. To calculate the diameter d of the emission region from the duration of the brightness variation we multiply the velocity of light (called c which equals $3 \times 10^8$ m/s), by the length of time (called $\Delta t$ or "delta t") it takes the AG to change brightness. When multiplied together they tell you the size of the emitting region:

$$\text{Size of emitting region} = \text{(speed of light)} \times \text{(time of variation)}$$

or

$$d = c\Delta t$$

Astronomers refer to this technique as using the light travel time ($\Delta t$) to find the size of an object ($c\Delta t$).

Example: A time variation of one week is observed. Determine the diameter of the AG's active region:

$$d = (3 \times 10^8 \text{ m/s}) \times (1 \text{ week})$$

Since 1 week = 7 days x 24 hrs/day x 3600 sec/hr = 604,800 seconds

$$d = (3 \times 10^8 \text{ m/s}) \times (604,800 \text{ seconds})$$

$$d = 1.81 \times 10^{14} \text{ m}$$

The solar system is roughly $10^{13}$ meters across, so the AG emitting region in this example is about 18 times the diameter of the solar system - an astoundingly small distance given that a typical AG can have a billion times the mass of our sun and about 1000 times the luminosity of the entire Milky Way galaxy!
There is a limit to this method. For example, if you only observe an object once a day, you won’t see variations shorter than that timescale. Therefore, the smallest region you can measure is 2 light days (about $5 \times 10^{13}$ meters, or five times the diameter of the solar system) across. The region is 2 light days across because you need to observe the object once, see its brightness rise, then see it fall. That takes three observations; the first, the second a day later, and the third a day after that: a two day interval. So when you measure the size based on brightness variations, it’s really an upper limit. There might be smaller scale regions, but you cannot detect them.

Also, the object may be emitting over various timescales: there may be short duration bursts or flares on top of much longer, slower variations. Generally speaking, astronomers assume that the shortest duration variation is what tells us the maximum size of the emitting region. The speed of light is the maximum speed the variation can travel across the object, but there may be slower methods too. So the quickest variation gives you an upper limit to the size of the region. It may be smaller than the calculated size, but not bigger.

In this exercise, the student takes data from a graph (see page 26) and uses it to estimate the total energy emitted by a flare every second. The flux from an object is defined as the amount of energy that passes through an area of one square meter every second. In the case of the graph of a flare from 3C279 (page 26), the plot shows the photon flux, which is the number of gamma-ray photons hitting one square meter every second. The graph is telling us that the flare generated about $1.75 \times 10^{2}$ photons per square meter per second. If we assume that each gamma ray hitting the detector has an energy of 1 billion electron Volts (abbreviated 1 GeV; about 1 billion times the energy of a visible light photon), the photon flux can be converted to an energy flux. That energy flux is the amount of energy hitting a square meter every second here on Earth. The Earth is at a vast distance from 3C279, so we only intersect a tiny fraction of the total energy emitted. If we could put a sphere around 3C279 such that the radius of the sphere is the distance from the Earth to the active galaxy, we would capture all the energy. If you add up all the energy in every square meter of the sphere, you will have the total energy emitted by the flare per second, what astronomers call the luminosity. The students will first use the graph to find the flux, then multiply it by the total area of that sphere, yielding the total luminosity of the flare. By dividing by the Sun's luminosity, they can directly compare the brightness of the flare to the brightness of the Sun. The ratio is huge, meaning that the flare emitted vastly more energy than the Sun does. Finally, they will calculate how long it would take the Sun to emit as much energy as the flare did in one second. The answer may surprise (and humble) you: the flare emitted about 125 trillion times as much energy as the Sun does every second, or the amount of energy emitted by the Sun over 4 million years!

Ask the students to think about that number for a moment. This was a single flare emitted by an object. The ratio they calculated was the energy of the flare in gamma rays only to the total energy of the Sun! In reality, even more energy was emitted by the flare, since the flare also emitted X-rays, ultraviolet, optical light, etc. Ask them if they would want to be anywhere near this object, and comment on what might happen if our own Milky Way Galaxy became an active galaxy.

Materials for each student:

- calculator
- pencil

Objectives:

Students will be able to use a simple equation to determine the size of a flare emitting region given the rise and decay times.

Students will be able to measure the rise and decay times of a flare using a plot with actual NASA data from an active galaxy.

Students will be able to compare the total energy emitted by the flare to the energy of the Sun.
Procedures:

1. Introduce the activity by reviewing information in the Introduction to Active Galaxies (page 5) and in the activity Background Information (page 22).

2. Discuss the speed of light with the students in terms of the distance covered in a given amount of time. How long would it take a beam of light to cross the Earth? Travel to the Moon? The Sun? The scale of the Universe is a very misunderstood topic; covering this topic before the activity will give the students a better sense of that scale. Ask them if a light year is a unit of distance or time. Many students will say "time," but the light year is a unit of length (equal to 9.5 x 10^{15} m).

3. Explain to the students that they will be doing two activities to measure the size and energy of a flaring region in an active galaxy. In the first activity, they will use a simple equation to determine the size of the region given the duration of a flare from that region. In the second they will measure the rise and decay times themselves using a plot with actual NASA data of an active galaxy. From there, they can compare the total energy emitted by the flare to the energy of the Sun.

4. The activities should be done individually.

Transfer Activities:

The idea that distance equals (speed x time) can be applied to many situations. For example, a car traveling at 100 kilometers per hour for two hours will travel a total of 200 kilometers. Have the students calculate the distance they can travel in a given time (say, 3 hours) given different vehicles: walking at 5 km/hr, bicycling at 30 km/hr, a car at 100 km/hr, a plane at 1000 km/hr, a satellite at 11 km/sec.

Extension Activities:

Do we see galaxies as they are “all at once,” or do we see some parts before others? A variation on the travel time idea is to use the delay between seeing an object and hearing a noise it makes to determine its distance. Imagine a lightning storm kilometers across, with two simultaneous bolts, one from the near side and the other from the far side. Do we hear the thunder at the same time? (No.) Do we see the bolts at exactly the same time? (No.) Is the delay between the two sounds the same as the delay between the two flashes? (No; the flashes are separated by milliseconds while the thunderclaps are separated by many seconds.) Given the speed of sound (~340 meters/second), have the students determine the distance to a lightning bolt given the delay in hearing the sound. (Each second of delay corresponds to 340 meters of distance.)

Reflection Activities:

How do dolphins use the differences in travel time in reflected sounds to see around them? Ask the students to think of where else this technique might be useful (radar, echo mapping of planet surface, distance to lightning storm).

Assessment:

There are two separate activities that comprise this unit. They can be assessed using the common rubric below either individually or as a group.

- **14 Points**
  - All calculations are correct

- **13 Points**
  - One calculation is incorrect, or off by more than 10%

- **12 Points**
  - Two calculations are incorrect

- **11 Points**
  - More than two calculations are incorrect

- **10 Points**
  - No work turned in
Answer Key for “Light Travel Time and the Size of Active Galaxies”:

Part A

<table>
<thead>
<tr>
<th>Time</th>
<th>Size (meters)</th>
<th>Size (solar system)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 hour</td>
<td>$1.1 \times 10^{12}$ meters</td>
<td>0.11 solar systems</td>
</tr>
<tr>
<td>1 day</td>
<td>$2.6 \times 10^{13}$ meters</td>
<td>2.6 solar systems</td>
</tr>
<tr>
<td>1 week</td>
<td>$1.8 \times 10^{14}$ meters</td>
<td>18 solar systems</td>
</tr>
<tr>
<td>1 year</td>
<td>$9.5 \times 10^{15}$ meters</td>
<td>950 solar systems</td>
</tr>
</tbody>
</table>

1) 4.7 seconds

2) 6 days
3) $1.6 \times 10^{14}$ meters
4) 16 solar systems
5) 6.8 days
6) $1.8 \times 10^{14}$ meters
7) 18 solar systems

Note: The answers to many of the following questions are based on the accuracy of hand-drawn measurements by the students. The answers in Part B assume the following:

a) a normal level of brightness for 3C279 to be $7.5 \times 10^{-3}$ photon m$^{-2}$ sec$^{-1}$
b) the brightness of the flare peaks at $17.5 \times 10^{-3}$ photon m$^{-2}$ sec$^{-1}$
c) the flare rise time begins at 5.5 days and ends at 11.5 days
d) the flare decay peak starts at $16.5 \times 10^{-3}$ photon m$^{-2}$ sec$^{-1}$

8) Since the same region is assumed to be getting brighter and then fading away, whichever time is faster should indicate the size of the region. The slower time indicates a larger region, but this larger region could not change in a shorter time. Therefore the faster time is a better indicator of the size of the region, which for this example is about $1.6 \times 10^{14}$ meters across. Another way to think about it: imagine the region is actually the size indicated by the slower change. That would mean it was 6.8 light days across. Would it be able to fade in 6 days? No, assuming it fades all at once, we would still see it as taking 6.8 days to fade. So it must actually be 6 light days across, and it took longer to fall in brightness due to some other physical effect.

9) Looking at the points for the minimum and maximum of the flare during its rise:
Slope = rise/run = $(17.5 \times 10^{-3}$ photon m$^{-2}$ sec$^{-1} – 7.5 \times 10^{-3}$ photon m$^{-2}$ sec$^{-1}$) / (11.5 days – 5.5 days) = $1.7 \times 10^{-3}$ photon m$^{-2}$ sec$^{-1}$ day$^{-1}$. To get the y-intercept, set y = mx + b and plug in the x and y for either the maximum or the minimum. The y-intercept is then $-1.7 \times 10^{-3}$ photon m$^{-2}$ sec$^{-1}$.

10) For the decay, rise/run = $(16.5 \times 10^{-3}$ photon m$^{-2}$ sec$^{-1} – 7.5 \times 10^{-3}$ photon m$^{-2}$ sec$^{-1}$) / (15 days – 21.8 days) = $-1.3 \times 10^{-3}$ photon m$^{-2}$ sec$^{-1}$ day$^{-1}$. The y-intercept is $36.5 \times 10^{-3}$ photon m$^{-2}$ sec$^{-1}$.

11) Add two weeks (14 days) to the time of maximum and put it into the equation for a line. x would then be 11.5 + 14 = 25.5, and y = mx + b, so y = 40.8 $\times 10^{-3}$ photon m$^{-2}$ sec$^{-1}$.

12) Since the rise time would be longer than the decay time, and the flare size depends on the shorter time, the flare size would be larger than that calculated from the original plot. (6.8 days is now the shorter of the two times.)

Part C

13) Maximum flux = $(17.5 \times 10^{-3}$ photon m$^{-2}$ sec$^{-1}) (1$ GeV/photon) = $17.5 \times 10^{-3}$ GeV m$^{-2}$ sec$^{-1}$.

14) Maximum flux = $(17.5 \times 10^{-3}$ GeV m$^{-2}$ sec$^{-1}) (10^9$ eV/GeV) / $(1.6 \times 10^{-19}$ eV/J) = $2.8 \times 10^{12}$ Joules m$^{-2}$ sec$^{-1}$.

15) Area = $4\pi[(4 \times 10^9 $light years)$ (9.5 \times 10^{15}$ m/light year)]$ = $1.8 \times 10^{52}$ m$^2$.

16) Area x flux = $(1.8 \times 10^{52}$ m$^2) (2.8 \times 10^{12}$ Joules m$^{-2}$ sec$^{-1}$) = $5 \times 10^{66}$ Joules/sec.

17) $(5 \times 10^{46}$Joules/sec) / $(4 \times 10^{28}$Joules/sec) = $1.25 \times 10^{18}$.

18) $1.25 \times 10^{18} / 3 \times 10^7 = 4.2 \times 10^4$ years or about 4.2 million years!
A. Finding the Size of an Active Galaxy Flare Region

1) Use the equation: \( \text{diameter} = c\Delta t = (3 \times 10^8 \text{ m/s}) \times \Delta t \) to determine the diameter of the active regions of AGs in meters and in units of the solar system diameter (use \( 10^{13} \) meters for the solar system diameter), for flares of duration \( \Delta t = 1 \) hour, 1 day, 1 week and 1 year.

<table>
<thead>
<tr>
<th>Time</th>
<th>Size (meters)</th>
<th>Size (solar system)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 hour</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 day</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 week</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 year</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2) Given the diameter of the sun (\( 1.4 \times 10^9 \) meters), what is the fastest the entire sun can vary in brightness, as measured in seconds?

**Question 1:** Time for solar variation (seconds): ______________

B. Measuring the Size of an Active Galaxy Flare Region

The plot at right is real astronomical data taken by the Large Area Telescope (LAT) aboard NASA’s Fermi Gamma-ray Space Telescope. The data were obtained in February 2009 and show flaring gamma-ray emission from the active galaxy 3C279. 3C279 is one of several dozen active galaxies that are regularly monitored by the LAT, as well as from optical telescopes on the ground.
There are two characteristic timescales for the variation shown. One is the “rise time,” or time it takes the brightness to reach its peak. The other is the “decay time,” or the time it takes for the brightness to drop back down to the normal level.

For the following exercise, assume the normal level of brightness for 3C279 is about $7.5 \times 10^{-3}$ photon m$^{-2}$s$^{-1}$.

Using only the data points where the flare is brightening, draw a “best-fit” line through the points. Do the same for the points where the flare is fading.

Use your hand-drawn line to determine the time duration ($\Delta t$) for the rise time of the flare. From that, calculate the size of the emitting region using $size = c\Delta t$ in both meters and in solar systems.

Using your hand-drawn line to the rising portion of the flare, determine the slope and intercept of the line ($y = mx + b$).

Do it again, but for the decaying portion of the flare.
If the flare had continued to rise for two more weeks, how bright would it have been?

Question 11: Brightness of flare after two more weeks (in the same units as the graph):

Question 12: Assuming the flare took two more weeks to rise than shown in the plot, but still decayed at the same rate, what does this imply about the size of the flaring region? Would it be bigger, smaller, or the same size as you found in the original plot?

The flux is the amount of energy hitting a square meter here on Earth. The total energy emitted by the flare every second is spread out over a sphere centered on the flare, with a radius equal to the distance from the Earth to the flare. The distance to 3C279 is about 4 x 10^9 light years, and there are 9.5 x 10^15 m in a light year. The surface area of a sphere is A = 4πr^2 where “r” is the radius of the sphere. What is the total surface area of the sphere?

Question 15: Surface area of sphere (m^2):

C: Measuring the Energy Emitted by an Active Galaxy Flare

From the graph, it’s possible to estimate the total energy emitted during the maximum of the flare and compare it to the energy emitted by the Sun. First, convert the units of the graph from photon flux to energy flux. Assume that each gamma ray hitting the detector has an energy of 1 GeV. Calculate the maximum flux in GeV m^-2 s^-1.

The total luminosity emitted in the flare each second, is given by E = (Area) x (Flux). Calculate the energy in Joules/sec.

Question 16: Total energy emitted per second by the flare (Joules/second):

The Sun’s luminosity is about 4 x 10^26 Joules/second. What is the ratio of the luminosity at the flare maximum to the solar luminosity?

Question 17: Total luminosity of the flare in Solar units:

There are about 3 x 10^7 seconds in a year. How many years would it take for the Sun to emit as much energy as the flare did in a single second?

Question 18: Number of years for the Sun to emit as much energy as the flare did in 1 second:
Definitions

**Accretion Disk**: the flattened disk of matter swirling just outside the black hole’s event horizon.

**Active Galaxy**: a galaxy with an unusually large amount of energy emitted from the core.

**Angular Size**: the angle between the lines of sight to the two opposite sides of an object.

**Annular**: shaped like or forming a ring.

**Apex**: the point or tip of the cone; a peak.

**Arcminute**: 1/60th of a degree.

**Black Hole**: an object so small and dense that inside its event horizon, the escape velocity is faster than the speed of light. In an active galaxy, the central black hole may have millions or even billions of times the Sun's mass.

**Blazar**: a quasar that one is viewing directly down the jet axis.

**Degree**: 1/360th of the circumference of a circle.

**Electron Volt (eV)**: a unit of energy commonly used in astronomy. A typical gamma ray has an energy of about 100 million electron Volts (100 MeV).

**Flare**: a sudden, dramatic increase in brightness of an object.

**Flux**: the amount of energy passing through an area of one square meter every second.

**Galaxy**: a collection of gas, dust, and billions of stars.

**Jet**: a thin, highly focused beam of matter and energy emitted from the cores of some active galaxies. Jets can be hundreds of thousands of light years long.

**Joule (J)**: a unit of energy, equal to $6.3 \times 10^{18}$ eV.

**Light-Week**: the distance light travels in a week; approximately 181 billion kilometers ($1.8 \times 10^{11}$ kilometers) or $1.8 \times 10^{14}$ m.

**Light-Year**: the distance light travels in one year; approximately 10 trillion kilometers ($10^{13}$ kilometers) or $9.5 \times 10^{15}$ m.
**Luminosity**: the total energy emitted by an object per second.

**Photon**: an individual quantum or particle of light.

**Quasar**: An active galaxy so distant it appears star-like.

**Quiescent**: at rest; inactive.

**Radian**: $1/(2\pi)$ of the circumference of a circle. One radian = 57.3 degrees

**Radio galaxy**: a galaxy that is a powerful source of radio waves.

**Radio Lobe**: A large radio-wave emitting cloud of matter located at the ends of the jets in some active galaxies, formed when the matter from the jet is slowed by intergalactic material.

**Seyfert galaxy**: an active galaxy (named for astronomer Carl Seyfert) where the inner disk is tipped to our line-of-sight, allowing us to see higher energy light from the nucleus.

**Solar System**: a collection of planets, moons, comets, etc. which orbits a star. Our solar system is roughly $10^{10}$ kilometers ($10^{13}$ meters) across.

**Torus**: A doughnut-shaped object. Gas and dust outside the accretion disk in an active galaxy orbit the central black hole in a torus-shaped region.

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**Acronyms**

**AG**: Active Galaxy

**E/PO**: Education and Public Outreach

**GBM**: Gamma-ray Burst Monitor

**GLAST**: Original, pre-launch, name for Fermi, which stood for Gamma-ray Large Area Space Telescope

**HST**: Hubble Space Telescope

**LAT**: Large Area Telescope (on Fermi)

**NASA**: National Aeronautics and Space Administration

**SSU**: Sonoma State University
National Science Education Standards:

Science as Inquiry

Activity 1:
- Abilities necessary to do scientific inquiry
  - Use of technology and mathematics to improve, invest, and communicate
  - Recognize and analyze alternative explanations and models
  - Design and conduct investigations
- Understanding about scientific inquiry

Activity 2:
- Abilities necessary to do scientific inquiry
  - Use of technology and mathematics to improve, invest, and communicate
  - Recognize and analyze alternative explanations and models
  - Design and conduct investigations
- Understanding about scientific inquiry

Activity 3:
- Abilities necessary to do scientific inquiry
  - Use of technology and mathematics to improve, invest, and communicate
  - Recognize and analyze alternative explanations and models
  - Design and conduct investigations
  - Identifying questions
- Understanding about scientific inquiry

Earth and Space Science

Activity 1:
- N/A

Activity 2:
- Properties and changes of properties in matter

Activity 3:
- Properties and changes of properties in matter
- Interactions of energy and matter

Physical Science

Activity 1:
- Origin and evolution of the universe

Activity 2:
- Origin and evolution of the universe

Activity 3:
- Origin and evolution of the universe

Science and Technology

Activity 1:
- Understanding about science and technology

Activity 2:
- Understanding about science and technology
  - Abilities of technological design
    - Implement a proposed solution
    - Evaluate the solution and its consequences
    - Communicate the problem, process, and solution

Activity 3:
- Understanding about science and technology
  - Abilities of technological design
    - Implement a proposed solution
    - Evaluate the solution and its consequences
    - Communicate the problem, process, and solution

History and Nature of Science

Activity 1:
- Science as a human endeavor
- Nature of science knowledge

Activity 2:
- Science as a human endeavor
- Nature of science knowledge

Activity 3:
- Science as a human endeavor
- Nature of science knowledge
Mathematics Standards for Active Galaxy Activities:

**Numbers and Operations**

*Activity 1:* N/A

*Activity 2:*
- judge the effects of such operations as multiplication, division, and computing powers and roots on the magnitudes of quantities
- develop fluency in operations with real numbers, vectors, and matrices, using mental computation or paper-and-pencil calculations for simple cases and technology for more-complicated cases.
- judge the reasonableness of numerical computations and their results.

*Activity 3:*
- judge the effects of such operations as multiplication, division, and computing powers and roots on the magnitudes of quantities
- develop fluency in operations with real numbers, vectors, and matrices, using mental computation or paper-and-pencil calculations for simple cases and technology for more-complicated cases.
- judge the reasonableness of numerical computations and their results.

**Algebra**

*Activity 1:*
- Use mathematical models to represent and understand quantitative relationships

*Activity 2:*
- Understand patterns, relations, and functions
- Represent and analyze mathematical situations and structures using algebraic symbols
- Use mathematical models to represent and understand quantitative relationships

*Activity 3:*
- Understand patterns, relations, and functions
- Represent and analyze mathematical situations and structures using algebraic symbols
- Use mathematical models to represent and understand quantitative relationships

**Geometry**

*Activity 1:*
- Specify locations and describe spatial relationships using coordinate geometry and other representational systems
- Use visualization, spatial reasoning, and geometric modeling to solve problems

*Activity 2:*
- Specify locations and describe spatial relationships using coordinate geometry and other representational systems
- Use visualization, spatial reasoning, and geometric modeling to solve problems

*Activity 3:* N/A

**Measurement**

*Activity 1:*
- Understand measurable attributes of objects and the units, systems, and processes of measurement

*Activity 2:*
- Understand measurable attributes of objects and the units, systems, and processes of measurement
- Apply appropriate techniques, tools, and formulas to determine measurements

*Activity 3:*
- Understand measurable attributes of objects and the units, systems, and processes of measurement
- Apply appropriate techniques, tools, and formulas to determine measurements

**Data Analysis and Probability**

*Activity 1:* N/A

*Activity 2:* N/A

*Activity 3:*
- Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them
- Develop and evaluate inferences and predictions that are based on data
For more information on Fermi and active galaxies, see the Fermi Education and Public Outreach web site:
http://fermi.sonoma.edu

This educational unit can be found on the Fermi Education and Public Outreach web site at:
http://fermi.sonoma.edu/teachers/index.php

Official NASA Fermi mission website:
http://www.nasa.gov/fermi

Compton Gamma-Ray Observatory and EGRET:
http://cossc.gsfc.nasa.gov/index.html

Fermi Project Site at Goddard Space Flight Center. This is the official site for Fermi Project Management at NASA’s Goddard Space Flight Center:
http://fermi.gsfc.nasa.gov/

The Large Area Telescope (LAT) Collaboration. The LAT is being managed by personnel at Stanford University. This is the official site for the LAT experiment:
http://www-glast.stanford.edu/

Gamma-ray Burst Monitor (GBM). The GBM is being managed by personnel at NASA’s Marshall Space Flight Center. This is the official site for the GBM experiment:
http://gammaray.msfc.nasa.gov/gbm/

The Center for Astrophysical Research in Antarctica has an interesting web page with small angle formula activities:
http://astro.uchicago.edu/cara/outreach/resources/ysi94/smallangle.html

A tutorial covering the discovery of quasars and how their size can be measured using $c\Delta t$:
http://casfso2.ucsd.edu/public/tutorial/Quasars.html

Bill Keel’s Active Galaxies page:
http://www.astr.ua.edu/keel/agn/

Ted Bunn’s Black Hole tutorial:
http://cosmology.berkeley.edu/Education/BHfaq.html
